Kolmogorov scaling in impassioned van Gogh paintings

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We show that some impassioned van Gogh paintings display scaling properties similar to the observed in turbulent fluids, suggesting that these paintings reflect the fingerprint of turbulence with such a realism that is even consistent with the way that a mathematical model characterizes this phenomenon. Specifically, we show that the probability distribution function (PDF) of luminance fluctuations of points (pixels) separated by a distance R is consistent with the Kolmogorov scaling theory in turbulent fluids. We also show that the most turbulent paintings of van Gogh coincide with periods of prolonged psychotic agitation of this artist.

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Everything in the last period of Vincent van Gogh paintings seems to be moving; this dynamical style served to transmit his own feelings about a figure or a landscape. It has been specifically mentioned that the famous painting *Starry Night*, vividly transmits the sense of turbulence and was compared with a picture of a distant star from the NASA/ESA Hubble Space Telescope, where eddies probably caused by dust and gas turbulence are clearly seen [1]. It is the purpose of this paper to show that some impassioned van Gogh paintings, painted during periods of prolonged psychotic agitation of this artist, reflect the fingerprint of turbulence with such a realism that is even consistent with the Kolmogorov scaling theory in turbulent fluids. Specifically, we show that the probability distribution function (PDF) of luminance fluctuations (δu) of points (pixels) separated by a distance R in *Starry Night*, and some other van Gogh paintings, is the same as the PDF of the velocity differences (δv) of pairs of points separated by a distance (R) in a turbulent flow as predicted by the statistical theory of Kolmogorov. This is not the first time that this scaling behavior is observed in a field far different from fluid mechanics; Kolmogorov scaling was also observed in fluctuations of the foreign exchange markets time series [2].

We mainly study van Gogh's *Starry Night* (June 1889), which undoubtedly transmits the feeling of turbulence. Also, as samples of another turbulent pictures, we analyze *Road with Cypress and Star* (May 1890) and *Wheat Field with Crows* (July 1890, just before van Gogh shot himself). By considering the analogy with the Kolmogorov scaling theory, from our results we can conclude that Vincent van Gogh was capable of capturing the fingerprint of turbulence. Our results also reinforce the idea that scientific objectivity may help to determine the fundamental content of artistic paintings, as was already done with Jackson Pollock's fractal paintings [3, 4]. Along this same ideas, it also worthy to mention that another notable ability of van Gogh was recently remarked with an experiment with bumblebees that had never seen natural flowers; insects were more attracted by van Gogh's *Sunflowers* than by other paintings containing flowers [5]. From this observation, Chittka and Walker suggest that van Gogh's flower paintings have captured the essence of floral features from a bee's point of view.

The statistical model of Kolmogorov [6, 7] is a foundation for modern turbulence theory. The main idea is that at very large Reynolds numbers, between the large scale of energy input (L) and the dissipation scale (η), at which viscous frictions become dominant, there is a myriad of small scales where turbulence displays universal properties independent of initial and boundary conditions. In particular, in the inertial range Kolmogorov predicts a famous scaling property of the second order structure function, $S_2(\mathbf{R}) = \langle (\delta v)^2 \rangle$, where $\delta v = \langle (v(\mathbf{r}+\mathbf{R}) - v(\mathbf{r}))^2 \rangle$ is the velocity increment between two points separated by a distance $\mathbf{R}$ and v is the component of the velocity in the direction of $\mathbf{R}$. In his first 1941 paper [6] Kolmogorov postulates two hypotheses of similarity that led to the prediction that $S_2(\mathbf{R})$ scales as $(\varepsilon R)^{2/3}$, where $R = \|\mathbf{R}\|$ and ε is the mean energy dissipation rate per unit mass. Under the same assumptions, in his third 1941 turbulence paper [7] Kolmogorov found an exact expression for the third moment, $\langle(\delta v)^3\rangle$, which is given by $S_3(\mathbf{R}) = -\frac{4}{5}\varepsilon R$. Furthermore, it has been claimed that this scaling results generalizes to structure functions of any order, *i.e.* $S_n(\mathbf{R}) = \langle(\delta v)^n\rangle \propto R^{\xi_n}$, where $\xi_n = n/3$. The PDF $P_R(\delta v)) = \delta v(R)/(\langle(\delta v(R))^2\rangle)^{1/2}$ is then scale invariant. Experimental measurements show that Kolmogorov was remarkably close to the truth in the sense that statistical quantities depend on the length scale R as a power law. The intermittent nature of turbulence causes, however, that the numerical values of ξ_n deviate progressively from $n/3$ when n increases, following a concave curve below the $n/3$ line [8].

Starry Night, painted during his one year period in the Saint Paul de Mausole Asylum at Saint-Rémy-de-Provence, is undoubtedly van Gogh's most mysterious masterpiece (Fig. 1). With the scene of a spectacularly transfigured sky, van Gogh immortalized his experience during a twilight state [9]. To perform the turbulence analysis of *Starry Night*, we start from a digitized, 300dpi, 2750×3542 image obtained from The Museum of Modern Art in New York (where the original paint lies), provided by Art Resource, Inc. We use the luminance (overall intensity) of the image since the eye is more sensitive to luminance changes than to color changes and usually most of the information about a scene is contained in its luminance. In a digital image, the luminance of a pixel is obtained from its RBG (red, green and blue) components as [10] $0.299R + 0.587G + 0.114B$. This approximate formula takes into account the fact that the human eye is more sensitive to green, then red and lastly blue. Thus, we use the PDF of pixel luminance fluctuations by building up a matrix whose rows contain difference in luminance δu and columns contain separation between pixels R. From this matrix, we determine the probability density of luminances $P_R(\delta u)$ with six pixel separations, $R = 60$, 240, 400, 600, 800, 1200, shown in Fig. 2. In order to rule out scaling artifacts, we have systematically recalculate the PDF function to images with lower resolutions (with an adequate rescaling of the pixel separations R). No significative differences appear up to images with resolutions lower that 150×127 pixels, where the details of the brushwork are lost.

By taking the analogy with Kolmogorov turbulence further, we take the large length scale as $L = 2000$ pixels, which is size of the largest eddy observed in the *Starry Night*. The cascade mechanism can be then corroborated if the statistical moments scale as $\langle(\delta u(R))^n\rangle \propto R^{\xi_n}$. In Fig. 3a, we show a log-log plot of the statistical moments with $n = 1$, 2, 3, 4, 5 (from bottom to top), which shows power-law scalings of the statistical moments with respect to the scale R. In each case straight line indicates the least-squares fit to the range of scales limited by the two dashed lines in the plot. In Fig. 3b, the scaling exponent ξ_n, of the first nine statistical moments are shown as a function of n. Data points are fitted to a straight line with slope 0.69 ± 0.0114 (with a 95% confidence bounds).

In turbulence, a functional form of the PDF at different scales has been parameterized with a Gaussian ansatz, using a model motivated by the cascade energy [11]. By superimposing several Gaussians at different scales, it is inferred that the shape of the PDF goes from nearly Gaussian at large scales R to nearly exponential at small scales. The number of superimposed Gaussians is controlled by a parameter, λ, which is the only parameter that must be fitted to the data. A large value of λ means that many scales contribute to the results, and thus the PDF develops tails that decay much slower than a pure Gaussian correlation. Curves of data points in Fig. 2 were fitted according to this model, yielding a notably good fit. Results are shown in the same figure with full lines; parameter values are $\lambda = 0.2$, 0.15, 0.12, 0.11, 0.09, 0.0009 (from bottom to top).

Kolmogorov's hypotheses of turbulence implies that the parameter λ^2 decreases linearly with $\ln(R)$. This also verifies indirectly the power-law scaling of the statistical moments and the validity of the cascade mechanism in Kolmogorov's turbulence picture. Figure 3(c) shows the dependence of λ^2 on R, the straight line indicates the least squares fit.

From van Gogh's 1890 period, we analyze two paintings: *Road with Cypress and Star* and *Wheat Field with Crows*. The former was painted just after the last and most prolonged psychotic episode of van Gogh's life, lasting from February to April 1890, during which the artist suffered terrifying hallucinations and severe agitation [9]. The later was painted just before the artist shot himself. Fig. 4 shows the PDF of *Road with Cypress and Star* and Fig. 5 shows the PDF of *Wheat Field with Crows*. In both cases the curves show the behaviour predicted by Kolmogorov's turbulence theory.

For comparison purposes, in Fig. 6 we show van Gogh's *Self-portrait with pipe and bandaged ear* and its PDF. In a well known episode of his life, on 23 December 1888, Vincent van Gogh mutilated the lower portion of his left ear. He was hospitalized at the Hôtel-Dieu hospital in Arles and prescribed potassium bromide [9]. After some weeks, van Gogh recovered from the psychotic state and, in a stage of absolute calm (as himself described in a letter to his brother Theo and sister Wilhemina [12]), he painted the self-portrait with pipe. As it can be seen in Fig. 6, the PDF of this paint departs from what is expected in Kolmogorov's model of turbulence

In summary, our results show that *Starry Night*, and other impassioned van Gogh paintings, painted during periods of prolonged psychotic agitation captured the essence of turbulence. We use Kolmogorov's model of turbulence to determine the degree of "realism" contained in the turbulent clouds of *Starry Night*. We are also suggesting new tools and approaches that open the possibility of quantitative objective research for art representation.

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Vincent van Gogh's *Starry Night*

www.moma.org/collection/printable_view.php?object_id=79802

FIG. 1: Vincent van Gogh's *Starry Night* (taken from the webpage of The Museum of Modern Art in New York.)

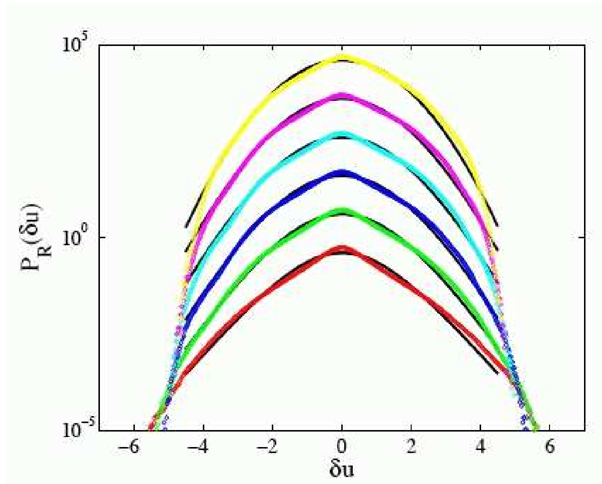

FIG. 2: Semilog plot of the probability density $P_R(\delta u)$ of luminance changes δu for pixel separations R = 60, 240, 400, 600, 800, 1200 (from bottom to top). Curves have been vertically shifted for better visibility. Data points were fitted, according to Ref. [11], and the results are shown in full lines; parameter values are λ = 0.2, 0.15, 0.12, 0.11, 0.09, 0.0009 (from bottom to top).

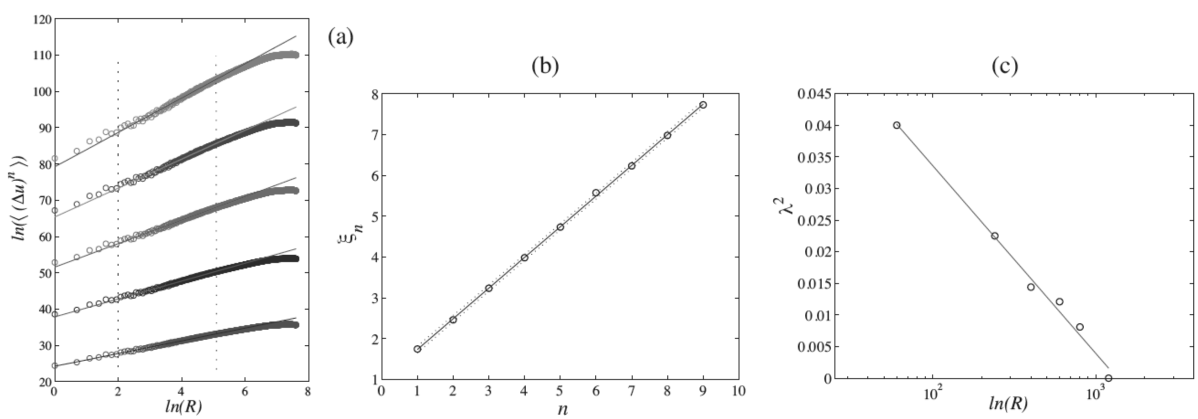

FIG. 3: (a) Log-log plot of the statistical moments $\langle(\delta u(R))^n\rangle$, with n = 1, 2, 3, 4, 5 (from bottom to top). The notation *Log* denotes a natural logarithm and in each case the straight line indicates the least-squares fit to the range of scales limited by the two dashed lines in the plot. (b) Exponent ξ_n of the statistical moments as a function of n. The straight line indicates the least-squares fit and dotted lines indicate the 95% confidence interval. (c) Dependence of λ^2 on R. Data points are fitted to a straight line by a least-square method.

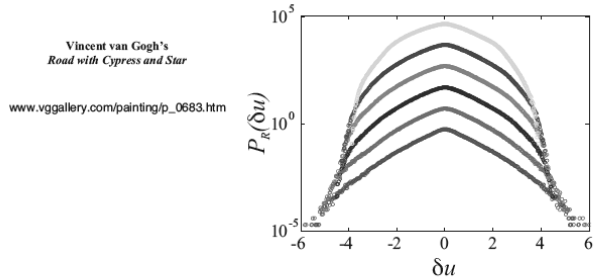

FIG. 4: Left: *Road with Cypress and Star* (Rijksmuseum Kröller-Müller, Otterlo). Right: PDF for pixel separations $R = 2$, 5, 15, 20, 30, 60 (from bottom to top). The image was taken from the WebMuseum-Paris, webpage.

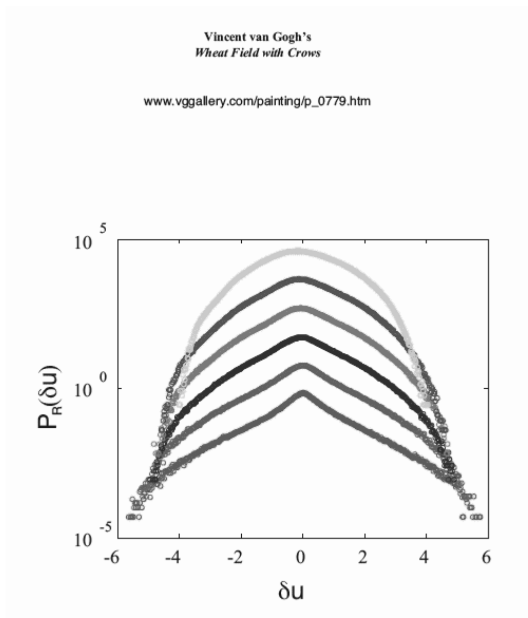

FIG. 5: *Wheat Field with Crows* (top) and its PDF (bottom) for pixel separations $R = 2$, 5, 15, 20, 30, 60 (from bottom to top). The image was taken from the Van Gogh Museum, Amsterdam, webpage.

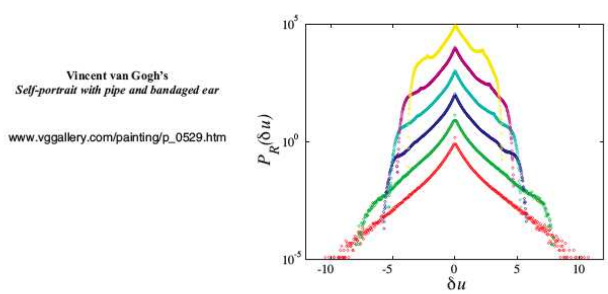

FIG. 6: *Self-portrait with pipe and bandaged ear* and its PDF. The image was taken from The Vincent van Gogh Gallery webpage.